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# Letters

## Comments on "A Rigorous Technique for Measuring the Scattering Matrix of a Multiport Device with a Two-Port Network Analyzer"

E. VAN LIL, MEMBER, IEEE

In the above paper,[1] Tippet and Speciale gave expressions for the correction to be made on the $S$ matrix to account for the mismatches on the ports not connected to the network analyzer.

The basic transformation was given by

$$S' = \left((I-S)^{-1}(I+S) - (I+\Gamma)(I-\Gamma)^{-1}\right) \cdot \left((I-S)^{-1}(I+S) + (I+\Gamma)(I-\Gamma)^{-1}\right)^{-1} \quad (1)$$

(notations as in the above paper[1] and [1]) from which the authors derived

$$S' = (I-S)^{-1}(S-\Gamma)(I-S\Gamma)^{-1}(I-S). \quad (2)$$

By using the relation

$$(I-A)^{-1}(I+A) = (I+A)(I-A)^{-1} \quad (3)$$

that can be easily proven by multiplying each side both right and left with $(I-A)$, we can rewrite (1) as

$$S' = \left((I+S)(I-S)^{-1} - (I-\Gamma)^{-1}(I+\Gamma)\right) \cdot \left((I+S)(I-S)^{-1} + (I-\Gamma)^{-1}(I+\Gamma)\right)^{-1}. \quad (4)$$

By following the same procedure as used in the derivation of (2) from (1), we obtain

$$S' = (I-\Gamma)^{-1}((I-\Gamma)(I+S) - (I+\Gamma)(I-S))(I-S)^{-1} \cdot (I-S)((I-\Gamma)(I+S) + (I+\Gamma)(I-S))^{-1}(I-\Gamma)$$

or

$$S' = (I-\Gamma)^{-1}(S-\Gamma)(I-\Gamma S)^{-1}(I-\Gamma) \quad (5)$$

proving the identity of (5) and (2) as was expected by Tippet and Speciale.

The simplification of Dropkin [1] applied by Tippet and Speciale to (5) gave

$$S' = (I+\Gamma)S(I-\Gamma S)^{-1}(I-\Gamma) - \Gamma \quad (6)$$

but does not mean a significant improvement in computational efficiency, because $I-\Gamma$, $\Gamma$, $I+\Gamma$, and $(I-\Gamma)^{-1}$ are diagonal matrices. So, (6) is only a little bit more efficient than (5) because it does not involve a division by $I-\Gamma$ but rather a multiplication by $I+\Gamma$. Furthermore, if a whole series of unknown $N$ ports has to be measured, the reflection coefficients in the diagonal matrix $\Gamma$ are known, so that only the computation of $S(I-\Gamma S)^{-1}$ has to be carried out, followed by a multiplication of column $i$ by $1-\Gamma_i$, row $j$ by $1+\Gamma_j$ and a subtraction of $\Gamma_k$ from diagonal element $k$. The formula by Dropkin [1], namely

$$S' = S - (I+S)\Gamma(I-S\Gamma)^{-1}(I-S) \quad (7)$$

even if it does contain a significant improvement over (2), it still is much less efficient than (6). Indeed, only the operation $(I+S)\Gamma$ or $\Gamma(I-S\Gamma)^{-1}$ can make use of the diagonal form of $\Gamma$. So, (6) gains a whole matrix multiplication and most of a matrix subtraction in computational effort over (7).

In the general case of an $N$-port measured with an $M$-port network analyzer, it is easy to show that (6) needs to be applied at most $N!/(M!(N-M)!)$ times for a $M \times M$ matrix and once

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[1] J. C. Tippet and R. A. Speciale, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 661–666, May 1982.

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to renormalize the $N \times N$ network to the 50-Ω system. This implies that each $S_{ij}$ with $i \neq j$ is measured $(N-2)!/((M-2)!(N-M)!)$ times (only once for the classical two-port network analyzer $(NA)(M=2)$), and each $S_{ii}(N-1)!/((M-1)!(N-M)!)$ times (or $(N-1)$ for the two-port $NA$). For the classical case of the two-port $NA$, the discussion about efficiency seems futile because one or two inversions of a $2 \times 2$ matrix will not make a large difference in computation time. However, for the renormalization to the 50-Ω system, the matrix size is $N \times N$.

The proof of (8) and (9) in the reply from the authors [1] is carried out in the same way as the simplified formulas (4) and (6) are derived from (1) and (3), but those formulas are only useful for simulation purposes.

*Reply*[2] *by J. C. Tippet and R. A. Speciale*[3]

The authors of the original paper[1] agree with Van Lil's proof of equivalence and with his analysis of the relative computational efficiencies of the four known forms of the generalized scattering matrix renormalization transform. Among these four forms, given by Van Lil as expressions (2), (5), (6), and (7), the first three were found by Speciale and the fourth by Dropkin.

For the record, Dropkin must be credited with providing the inspiration that stimulated the derivation of (6) from (5) and for delivering, already in December 1982, a direct proof of equivalence of (6) to (7). We are reporting this proof in full at the end of this reply, as it is, to our knowledge, still unpublished. The proof of equivalence of (2) to (5) delivered by Van Lil is, however, original, and he must be credited for that.

We are, however, sorry to have to disagree with Van Lil's conclusions relating to the minimum number of partial scattering measurements required to fully characterize an $N$-port network on an $M$-port network analyzer. Our disagreement is motivated by the following counter-example: Only three partial measurements are required to fully characterize a 6-port network on a 4-port network analyzer. One possible strategy is to use the port-combinations (1, 2, 3, 4), (1, 2, 5, 6), and (3, 4, 5, 6) in which case three $4 \times 4$ preliminary renormalizations are required prior to the final $6 \times 6$ renormalization. Also, each $S_{ii}$ is measured twice while all $S_{ij}$ are measured once except for the $S_{12}$, $S_{21}$, $S_{34}$, $S_{43}$, $S_{56}$, and $S_{65}$ entries which are measured twice. This example supports the conjecture, stated in a footnote of our paper[1] that $N(2N-M)/M^2$ is the minimum number of required partial measurements, not $N!/(M!(N-M)!)$ as stated by Van Lil.

The above conjecture only applies to a situation where $M$ is even and $N$ is a multiple of $M/2$. It would be interesting to find a proof of this conjecture and possibly an expression applicable to arbitrary $N$ and $M$ values. Another interesting aspect of this problem is to find a formal method for selecting which sets of port-combinations attain the minimum number of partial measurements. Indeed, even in the above counter-example, there are various alternate sets that attain full characterization of a 6-port network in the minimum number of three partial measurements.

Finally, we would like to observe that the application of the generalized renormalization transform to the measurement problem described in [1] is not the only one. Another interesting application is the prediction of the true scattering response of a multiport network in its intended system environment, where the impedances seen by the various ports are, in general, far from the nominal impedance used in basic design and testing. In fact, we envisage some kind of "reverse-design" procedure where multiport system components would be designed to meet given scattering response specifications in a specified, non-nominal, external port-impedance environment, rather than in a nominal-impedance environment. System components would thus be designed to fit very specific "niches" and would be tested against substantially different reference scattering responses, normalized to nominal external port-impedances at all ports. Such reference responses would obviously be specified through renormalization of the required response from the true-environment impedances to the nominal impedances.

The direct proof of equivalence of (6) to (7) delivered by Dropkin in December 1982 was formulated as follows:

$$S_1' = S - (I+S)\Gamma(I - S\Gamma)^{-1}(I-S)$$

is the same as the alternate form

$$S_2' = (I+\Gamma)S(I-\Gamma S)^{-1}(I-\Gamma) - \Gamma.$$

Note first that $S(I-\Gamma S)^{-1} = (I-S\Gamma)^{-1}S$ and that if we set $A = (I-S\Gamma)^{-1}$, then $(S\Gamma)$ commutes with $A$

$$\begin{aligned} S_1' - S_2' &= S - (I+S)\Gamma A(I-S) - (I+\Gamma)AS(I-\Gamma) + \Gamma \\ &= S + \Gamma \underset{(1)}{- \Gamma A} \underset{(2)}{- S\Gamma A} \underset{(3)}{+ \Gamma AS} \underset{(4)}{+ S\Gamma AS} \underset{(5)}{- AS} \\ &\quad \underset{(6)}{+ AS\Gamma} \underset{(7)}{- \Gamma AS} \underset{(8)}{+ \Gamma AS\Gamma} \end{aligned}$$

$$3+7=0$$

$$1+8 = -\Gamma A + \Gamma AS\Gamma = -\Gamma A(I - S\Gamma) = -\Gamma$$

$$4+5 = S\Gamma AS - AS = (S\Gamma - I)AS = -S$$

$$2+6 = -S\Gamma A + AS\Gamma = 0 \text{ since } A \text{ commutes with } S\Gamma$$

$$S_1' - S_2' = 0$$

$$S_1' = S_2'.$$

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[2] Manuscript received September 26, 1984

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## Corrections to "High-Temperature Microwave Characterization of Dielectric Rods"

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In the above paper,[1] the fourth and fifth sentences in the third paragraph from the bottom of the right-half of p. 1332, should read:

"The bisection method is used twice; once to find the roots of $G(\beta)$, and secondly to find the roots of $F(\beta)$. In

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